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Gazeau–Klauder squeezed states associated with solvable quantum systems

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Abstract

A formalism for the construction of some classes of Gazeau–Klauder squeezed states, corresponding to arbitrary solvable quantum systems with a known discrete spectrum, are introduced. As some physical applications, the proposed structure is applied to a few known quantum systems and then statistical properties as well as squeezing of the obtained squeezed states are studied. Finally, numerical results are presented.

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1. Introduction

The standard coherent states may be obtained from the action of the 'displacement operator' (or 'coherence operator') on the vacuum:

$$D(z) = \exp(za^\dagger - z^*a), \qquad D(z)|0\rangle = |z\rangle, \tag{1}$$

where a and $a^\dagger$ are the standard bosonic annihilation and creation operators, respectively. Nowadays, generalization of coherent states and their experimental generations have made quantum physics very interesting, especially quantum optics [1]. These states exhibit some interesting '*nonclassical properties*' particularly quadrature squeezing, antibunching, sub-Poissonian photon statistics and oscillatory number distribution.

Although some classes of generalized coherent states may possess squeezing in one of the quadrature components of the radiation field, another set of states known as '*squeezed states*' which are the simplest representatives of nonclassical states also play an important role in quantum optics. These states are nonclassical states of the electromagnetic radiation field in which certain observables exhibit fluctuations less than the vacuum. Among them, the 'standard squeezed states' are obtained by the action of a 'squeezing operator' on the vacuum [22]:

$$S(\xi) = \exp\left[\tfrac{1}{2}\left(\xi a^{\dagger 2} - \xi^* a^2\right)\right], \qquad S(\xi)|0\rangle = |\xi\rangle. \tag{2}$$

Bearing in mind that a special realization of the $su(1,1)$ algebra can be considered with the generators $K_0 = \frac{1}{2}\left(a^\dagger a + \frac{1}{2}\right)$, $K_+ = \frac{1}{2}(a^\dagger)^2$, $K_- = \frac{1}{2}a^2$, the squeezed operator $S(\xi)$ in

(2) may be rewritten as the displacement operator $D(\xi) = \exp\left[\frac{1}{2}(\xi K_+ - \xi^* K_-)\right]$. These are the Perelomov [16] form of generalized coherent states which defined for a reducible representation of the $SU(1,1)$ group in analogy with the displacement operator definition. Therefore, the squeezed states in (2) have been sometimes called the $SU(1,1)$ coherent states, which are of great relevance to quantum optics for single-mode fields. The number states expansion of the states in (2) will be reobtained as a special case of the proposed formalism in the present paper (see example 1 in section 4). Anyway, squeezing (of the quantized radiation field) means that the uncertainty of one of the quadratures of the field falls below the uncertainty of the vacuum state at the cost of increased uncertainty in the other quadrature; hence, the state is nonclassical. The usefulness of this property in various fields such as the measurement techniques and detection of gravitational waves [5], enhancement and suppression of spontaneous emission [4] and optical communication [21] is well understood. Some generalizations of these states are also introduced in the literature. To name a few, it may be referred to as '*squeezed coherent states*', '*nonlinear squeezed states*' [11] '*representations of squeezed states in an f-deformed Fock space*' [17] and '*a class of nonlinear squeezed states*', recently introduced in [15] (for a squeezed review on nonclassical states, see [6] and references therein).

The present work is motivated to enlarge the class of squeezed states and especially provide a framework to be able to introduce the 'squeezed states' in a direct relation to physical systems. Recall that this purpose has been achieved in the 'generalized coherent states' domain by J-P Gazeau and J R Klauder in an elegant fashion [8–10]. Actually, in addition to the '*continuity*' and the '*resolution of the identity*', they imposed '*temporal stability*' and '*action identity*' requirements as two new physical criteria to force the generalized coherent states to a more classical situation. Obviously, any (coherent or squeezed) state preserves the temporal stability under the Hamiltonian dynamics if one considers the eigenvalue equation $\hat{H}|n\rangle = n|n\rangle$, i.e. that of a harmonic oscillator. Whereas, if one deals with a specific quantum system with the Hamiltonian $\hat{\mathcal{H}}$ and eigenenergies e_n, so that $\hat{\mathcal{H}}|n\rangle = e_n|n\rangle$, and looks forward to construct the coherent or squeezed states, there will appear some problems with the temporal stability of the states in hand. Fortunately, the proposed formalism of Gazeau and Klauder attacked the problem, and the so-called '*Gazeau–Klauder coherent states*' possess the temporal stability property, under the action of the time evolution operator $\mathrm{e}^{-\mathrm{i}\hat{\mathcal{H}}t}$, essentially. The Hamiltonian $\hat{\mathcal{H}}$ in the latter operator is responsible for the dynamics of the quantum system. As Roknizadeh *et al* algebraically established in [18, 19], the Hamiltonian $\hat{\mathcal{H}}$ is constructed by $A_{\mathrm{GK}} = af_{\mathrm{GK}}(\alpha, \hat{n})$ and $A^\dagger = f^\dagger_{\mathrm{GK}}(\alpha, \hat{n})a^\dagger$ as the f-deformed annihilation and creation operators, respectively, via the factorization method $\hat{\mathcal{H}} = A^\dagger_{\mathrm{GK}}A_{\mathrm{GK}}$. In this fashion, the Gazeau–Klauder coherent states are the generalized nonlinear coherent states with the generalized operator-valued nonlinearity function $f_{\mathrm{GK}}(\alpha, \hat{n})$, which depends explicitly on the intensity of light.

Finally, a point is worth mentioning. Recall that in the construction of the dual pair of the Gazeau–Klauder coherent states in [19], a different and non-usual method has been employed. This was due to recognition that the dual pair of the Gazeau–Klauder coherent states must possess all of the four mentioned criteria in [8], carefully. But, as will be demonstrated in the continuation of the present paper, although the presented formalism is essentially based on the structure of the Gazeau–Klauder coherent states, it is not necessary to reconsider the whole criteria of Gazaeu and Klauder for the squeezed states will be introduced in this paper. The reason is clear since when one deals with the squeezed states, he (she) automatically relaxes from classicality.

This paper is organized as follows. Section 2 is devoted to a brief review on the fundamental structure of Gazeau–Klauder coherent states. A new (and more general) proposal for constructing a set of squeezed states, which have been called 'Gazeau–Klauder squeezed states', will be presented in section 3. Then, in section 4 the formalism will be applied to some quantum systems with a known discrete spectrum, and finally in section 5 the quantum statistical properties and squeezing of the obtained states will be studied.

2. Gazeau–Klauder coherent states as nonlinear coherent states

Gazeau–Klauder coherent states have been introduced associated with quantum systems with a known discrete spectrum E_n [8]. According to [7, 8], the analytical representations of these states have been introduced as follows:

$$|z,\alpha\rangle \doteq \mathcal{N}(|z|^2)^{-1/2}\sum_{n=0}^{\infty}\frac{z^n\,\mathrm{e}^{-\mathrm{i}\alpha e_n}}{\sqrt{\rho(n)}}|n\rangle, \qquad z\in\mathbb{C}, \quad 0\neq\alpha\in\mathbb{R}, \tag{3}$$

where $\mathcal{N}(|z|^2)$ is some appropriate normalization constant. In equation (3), the kets $\{|n\rangle\}_{n=0}^{\infty}$ are the eigenvectors of the Hamiltonian $\hat{\mathcal{H}}$ with the eigenenergies E_n such that

$$\hat{\mathcal{H}}|n\rangle = E_n|n\rangle \equiv \hbar\omega e_n|n\rangle \equiv e_n|n\rangle, \qquad \hbar = 1 = \omega, \qquad n = 0, 1, 2, \ldots, \tag{4}$$

where the rescaled spectrum e_n satisfies the inequalities $0 = e_0 < e_1 < e_2 < \cdots < e_n < e_{n+1} < \cdots$. The action identity criteria under the condition $e_0 = 0$ imposed the requirement $\rho(n) = [e_n]!$.

It is established in [18] that the states in expansion (3) are '*nonlinear coherent states*' with the operator-valued (and also intensity-dependent) nonlinearity function

$$f_{\mathrm{GK}}(\alpha,\hat{n}) = \mathrm{e}^{\mathrm{i}\alpha(\hat{e}_n-\hat{e}_{n-1})}\sqrt{\frac{\hat{e}_n}{\hat{n}}}, \qquad \hat{e}_n = \frac{\rho(\hat{n})}{\rho(\hat{n}-1)}, \tag{5}$$

where $\hat{n} = a^\dagger a$ is the number operator. The explicit dependence of the nonlinearity function on the spectrum of an arbitrary quantum system is notable. Therefore, the rising and lowering operators related to any solvable system may be defined as

$$A_{\mathrm{GK}} = a f_{\mathrm{GK}}(\alpha,\hat{n}), \qquad A_{\mathrm{GK}}^\dagger = f_{\mathrm{GK}}^\dagger(\alpha,\hat{n})a^\dagger, \tag{6}$$

with a commutator between A_{GK} and $A_{\mathrm{GK}}^\dagger$ as

$$\left[A_{\mathrm{GK}}, A_{\mathrm{GK}}^\dagger\right] = (\hat{n}+1)f_{\mathrm{GK}}(\hat{n}+1)f_{\mathrm{GK}}^\dagger(\hat{n}+1) - \hat{n}f_{\mathrm{GK}}^\dagger(\hat{n})f_{\mathrm{GK}}(\hat{n}). \tag{7}$$

Equations (5) and (6) clearly show that the f-deformed ladder operators for any solvable quantum system may be easily obtained. Using the '*normal-ordered*' form of the Hamiltonian and taking $\hbar = 1 = \omega$, for the Hamiltonian corresponding to the Gazeau–Klauder coherent states, one gets

$$\hat{\mathcal{H}} \equiv A_{\mathrm{GK}}^\dagger A_{\mathrm{GK}} = \hat{n}|f_{\mathrm{GK}}(\alpha,\hat{n})|^2 \equiv \hat{e}_n. \tag{8}$$

Consequently, relation (4) holds, obviously.

Moreover, one can get the two canonical conjugates of the operators A_{GK} and $A_{\mathrm{GK}}^\dagger$, as $B_{\mathrm{GK}}^\dagger = \frac{1}{f_{\mathrm{GK}}^\dagger(-\alpha,\hat{n})}a^\dagger$ and $B_{\mathrm{GK}} = a\frac{1}{f_{\mathrm{GK}}(-\alpha,\hat{n})}$, respectively. As a result $\left[A_{GK}, B_{GK}^\dagger\right] = \hat{I} = \left[B_{GK}, A_{GK}^\dagger\right]$, where $\hat{I}$ is the unit operator. Roknizadeh *et al* have introduced these sets of operators to establish that the Gazeau–Klauder coherent states may be constructed by a non-unitary displacement-type operator [19]:

$$D(z)|0\rangle = \exp\left(zA_{\mathrm{GK}}^\dagger - z^* B_{\mathrm{GK}}\right)|0\rangle = |z,\alpha\rangle. \tag{9}$$

It is also shown that another displacement operator may be introduced by which one can derive another class of nonlinear coherent states (which have been called the '*dual states*' [2, 19]) as follows:

$$\tilde{D}(z)|0\rangle = \exp\left(zB_{\rm GK}^{\dagger} - z^{*}A_{\rm GK}\right)|0\rangle = |\widetilde{z,\alpha}\rangle. \tag{10}$$

For achieving this purpose, recall that the 'dual family of the Gazeau–Klauder coherent states' has been introduced in [19] as follows:

$$|\widetilde{z,\alpha}\rangle \doteq \mathcal{N}(|z|^2)^{-1/2}\sum_{n=0}^{\infty}\frac{z^n\,{\rm e}^{-{\rm i}\alpha\varepsilon_n}}{\sqrt{\mu(n)}}|n\rangle, \qquad \mu(n) = \frac{(n!)^2}{\rho(n)}, \qquad z\in\mathbb{C}, \quad 0\neq\alpha\in\mathbb{R}. \tag{11}$$

The deformed annihilation and creation operators $\widetilde{A}_{\rm GK}$ and $\widetilde{A}_{\rm GK}^{\dagger}$ of the dual oscillator algebra encountered the operator-valued nonlinearity function

$$\widetilde{f}_{\rm GK}(\alpha,\hat{n}) = {\rm e}^{{\rm i}\alpha(\hat{\varepsilon}_n - \hat{\varepsilon}_{n-1})}\sqrt{\frac{\hat{\varepsilon}_n}{\hat{n}}}, \qquad \varepsilon_n \equiv \frac{\mu(n)}{\mu(n-1)}. \tag{12}$$

Hence, the deformed annihilation and creation operators of the dual oscillator may be expressed as

$$\widetilde{A}_{\rm GK} = a\widetilde{f}_{\rm GK}(\alpha,\hat{n}), \qquad \widetilde{A}_{\rm GK}^{\dagger} = \widetilde{f}_{\rm GK}{}^{\dagger}(\alpha,\hat{n})a^{\dagger}. \tag{13}$$

The normal-ordered Hamiltonian of the dual oscillator is therefore

$$\widetilde{\mathcal{H}} = \widetilde{A}_{\rm GK}^{\dagger}\widetilde{A}_{\rm GK} = \hat{n}|\widetilde{f}_{\rm GK}(\alpha,\hat{n})|^2 \equiv \hat{\varepsilon}_n. \tag{14}$$

As a result,

$$\widetilde{\mathcal{H}}|n\rangle = \varepsilon_n|n\rangle, \qquad \varepsilon_n \equiv \widetilde{e}_n = \frac{n^2}{e_n}, \tag{15}$$

where again the units $\omega = 1 = \hbar$ have been used. The eigenvalues of $\widetilde{\mathcal{H}}$ are also required to satisfy the following inequalities: $0 = \varepsilon_0 < \varepsilon_1 < \varepsilon_2 < \cdots < \varepsilon_n < \varepsilon_{n+1} < \cdots$.

Now constructing the two conjugate operators of $\widetilde{A}_{\rm GK}$ and $\widetilde{A}_{\rm GK}^{\dagger}$, i.e. $\widetilde{B}_{\rm GK} = a\frac{1}{\widetilde{f}_{\rm GK}(-\alpha,\hat{n})}$ and $\widetilde{B}_{\rm GK}^{\dagger} = \frac{1}{\widetilde{f}_{\rm GK}^{\dagger}(-\alpha,\hat{n})}a^{\dagger}$ respectively, one has $[\widetilde{B}_{\rm GK},\widetilde{A}_{\rm GK}^{\dagger}] = \hat{I} = [\widetilde{A}_{\rm GK},\widetilde{B}_{\rm GK}^{\dagger}]$.

Substituting $\alpha = 0$, so that $e_n = nf^2(n)$ $\left(\varepsilon_n = \frac{n}{f^2(n)}\right)$, in the above relations (equations (3) to (15)) in this section will recover the nonlinear coherent states (their dual family) which have been already introduced by Man'ko *et al* [12], de Matos Filho *et al* [13, 14] (Ali *et al* [2] and Roy *et al* [20]).

3. General structure of Gazeau–Klauder squeezed states

Following the explained path for the introduction of the dual pair of the Gazeau–Klauder coherent states, '*dual families of the Gazeau–Klauder squeezed states*' for arbitrary solvable quantum systems with a known discrete spectrum can be simply obtained. Upon using the considerations outlined in section 2, two new classes of squeezed states may be introduced by the actions of the '*generalized squeezing operators*' S and $\widetilde{S}$, which are now '*energy dependent*', on the vacuum as follows:

$$S(\xi,\alpha,f)|0\rangle = \exp\left[\tfrac{1}{2}\left(\xi A_{\rm GK}^{\dagger\,2} - \xi^{*}B_{\rm GK}^{2}\right)\right]|0\rangle = |\xi,\alpha,f\rangle, \tag{16}$$

and

$$\tilde{S}(\xi,\alpha,f)|0\rangle = \exp\left[\tfrac{1}{2}\left(\xi B_{\rm GK}^{\dagger\,2} - \xi^{*}A_{\rm GK}^{2}\right)\right]|0\rangle = |\widetilde{\xi,\alpha,f}\rangle. \tag{17}$$

The explicit form of the generalized squeezed states, $|\xi, \alpha, f\rangle$, may be straightforwardly found by the superposition of even Fock states:

$$|\xi, \alpha, f\rangle = \mathcal{N} \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{n!} [f_{\mathrm{GK}}^{\dagger}(\alpha, 2n)]! \left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n |2n\rangle, \tag{18}$$

where in this case and what follows, $\xi = \tanh r \exp(\mathrm{i}\phi)$ and $\mathcal{N}$ is chosen so that the states be normalized. Inserting the explicit form of the nonlinearity function f_{GK} from (5) with the help of the definition of Jackson's factorial, one immediately gets $\left[f_{\mathrm{GK}}^{\dagger}(\pm\alpha, n)\right]! = \mathrm{e}^{\mp \mathrm{i}\alpha e_n}\sqrt{\frac{[e_n]!}{n!}}$. To this end, the explicit form of the first class of the '*Gazeau–Klauder squeezed states*' will be expressed as the following superposition:

$$|\xi, \alpha, f\rangle = \mathcal{N} \sum_{n=0}^{\infty} \mathrm{e}^{-\mathrm{i}\alpha e_{2n}} \frac{\sqrt{[e_{2n}]!}}{n!} \left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n |2n\rangle, \tag{19}$$

where

$$\mathcal{N} = \left[\sum_{n=0}^{\infty} \frac{[e_{2n}]!}{(n!)^2} \left(\frac{\tanh r}{2}\right)^{2n}\right]^{-\frac{1}{2}}. \tag{20}$$

The states $|\widetilde{\xi, \alpha, f}\rangle$ introduced previously in (17), the dual pair of the generalized squeezed states in (16), may be given by the superposition of the even Fock states:

$$|\widetilde{\xi, \alpha, f}\rangle = \mathcal{N}' \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{n!} \frac{1}{\left[f_{\mathrm{GK}}^{\dagger}(-\alpha, 2n)\right]!} \left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n |2n\rangle. \tag{21}$$

Again, inserting the equivalent form of $\left[f_{\mathrm{GK}}^{\dagger}(-\alpha, n)\right]!$ in terms of the eigenvalues of the system e_n, the '*dual family of the Gazeau–Klauder squeezed states*' can be rewritten in the form

$$|\widetilde{\xi, \alpha, f}\rangle = \mathcal{N}' \sum_{n=0}^{\infty} \mathrm{e}^{-\mathrm{i}\alpha e_{2n}} \frac{(2n)!}{n!\sqrt{[e_{2n}]!}} \left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n |2n\rangle, \tag{22}$$

where

$$\mathcal{N}' = \left[\sum_{n=0}^{\infty} \left(\frac{(2n)!}{n!}\right)^2 \frac{1}{[e_{2n}]!} \left(\frac{\tanh r}{2}\right)^{2n}\right]^{-\frac{1}{2}}. \tag{23}$$

In addition to the above two distinct classes of squeezed states (19) and (22) which are in duality, it is also possible to propose two new sets of squeezed states based on the dual Hamiltonian in (14) as follows:

$$S(\xi, \alpha, \widetilde{f})|0\rangle = \exp\left[\tfrac{1}{2}\left(\xi\left(\widetilde{A}_{\mathrm{GK}}^{\dagger}\right)^2 - \xi^* \widetilde{B}_{\mathrm{GK}}^2\right)\right]|0\rangle = |\xi, \alpha, \widetilde{f}\rangle, \tag{24}$$

and

$$\tilde{S}(\xi, \alpha, \widetilde{f})|0\rangle = \exp\left[\tfrac{1}{2}\left(\xi\left(\widetilde{B}_{\mathrm{GK}}^{\dagger}\right)^2 - \xi^* \widetilde{A}_{\mathrm{GK}}^2\right)\right]|0\rangle = |\widetilde{\xi, \alpha, \widetilde{f}}\rangle. \tag{25}$$

The generalized squeezed states, in (24) and (25), can be decomposed by the even Fock states as

$$|\xi, \alpha, \widetilde{f}\rangle = \widetilde{\mathcal{N}} \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{n!} [\widetilde{f}_{\mathrm{GK}}^{\dagger}(\alpha, 2n)]! \left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n |2n\rangle, \tag{26}$$

and

$$|\widetilde{\xi, \alpha, \widetilde{f}}\rangle = \widetilde{\mathcal{N}}' \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{n!} \frac{1}{\left[\widetilde{f}_{\mathrm{GK}}^{\dagger}(-\alpha, 2n)\right]!} \left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n |2n\rangle. \tag{27}$$

Substituting the explicit form of $\left[\widetilde{f}^{\dagger}_{\rm GK}(\pm\alpha, n)\right]! = {\rm e}^{\pm {\rm i}\alpha\varepsilon_n}\sqrt{\frac{[\varepsilon_n]!}{n!}}$, in (26) and (27), one readily obtains the third and fourth classes of the *Gazeau–Klauder squeezed states* with the following superpositions:

$$|\xi, \alpha, \widetilde{f}\rangle = \widetilde{\mathcal{N}} \sum_{n=0}^{\infty} {\rm e}^{-{\rm i}\alpha\varepsilon_{2n}} \frac{\sqrt{[\varepsilon_{2n}]!}}{n!} \left[\frac{\exp({\rm i}\phi)\tanh r}{2}\right]^n |2n\rangle, \tag{28}$$

and

$$\widetilde{|\xi, \alpha, \widetilde{f}\rangle} = \widetilde{\mathcal{N}}' \sum_{n=0}^{\infty} {\rm e}^{-{\rm i}\alpha\varepsilon_{2n}} \frac{(2n)!}{n!\sqrt{[\varepsilon_{2n}]!}} \left[\frac{\exp({\rm i}\phi)\tanh r}{2}\right]^n |2n\rangle, \tag{29}$$

where $\widetilde{\mathcal{N}}$ and $\widetilde{\mathcal{N}}'$ may be determined by the normalization condition as follows:

$$\widetilde{\mathcal{N}} = \left[\sum_{n=0}^{\infty} \frac{[\varepsilon_{2n}]!}{(n!)^2} \left(\frac{\tanh r}{2}\right)^{2n}\right]^{-\frac{1}{2}}, \tag{30}$$

and

$$\widetilde{\mathcal{N}}' = \left[\sum_{n=0}^{\infty} \left(\frac{(2n)!}{n!}\right)^2 \frac{1}{[\varepsilon_{2n}]!} \left(\frac{\tanh r}{2}\right)^{2n}\right]^{-\frac{1}{2}}. \tag{31}$$

respectively. The introduced states in (19) and (22) (in (28) and (29)) explicitly show the relation of the Gazeau–Klauder squeezed states (the dual of the Gazeau–Klauder squeezed states) to the spectrum of the quantum system (the dual of the quantum system). Note that from the form of the four classes of obtained squeezed states in equations (19), (22), (28) and (29), it may be recognized that they are temporally stable, i.e. possess one of the main features of the Gazeau–Klauder coherent states. This is in fact due to the existence of the exponential term $\exp(\pm {\rm i}\alpha\epsilon_n)$ in the expansion coefficient of the obtained squeezed states (ϵ stands appropriately for e or ε). Hence, for instance by the following definition, the invariance of the squeezed states under the appropriate time evolution operator can be guaranteed, i.e.

$$ {\rm e}^{{\rm i}\hat{\mathcal{H}}t}|\xi, \alpha, f\rangle = |\xi, \alpha + {\rm i}\omega t, f\rangle, \qquad {\rm e}^{{\rm i}\hat{\mathcal{H}}t}\widetilde{|\xi, \alpha, f\rangle} = \widetilde{|\xi, \alpha + {\rm i}\omega t}, f\rangle, \tag{32}$$

$$ {\rm e}^{{\rm i}\hat{\widetilde{\mathcal{H}}}t}|\xi, \alpha, \widetilde{f}\rangle = |\xi, \alpha + {\rm i}\omega t, \widetilde{f}\rangle, \qquad {\rm e}^{{\rm i}\hat{\widetilde{\mathcal{H}}}t}\widetilde{|\xi, \alpha, \widetilde{f}\rangle} = \widetilde{|\xi, \alpha + {\rm i}\omega t}, \widetilde{f}\rangle. \tag{33}$$

To investigate the above equations, there should be emphasis on using the eigenvalue equations $\hat{\mathcal{H}}|n\rangle = e_n|n\rangle$ in (32) and $\hat{\widetilde{\mathcal{H}}}|n\rangle = \varepsilon_n|n\rangle$ in (33). Therefore, seemingly the name '*temporally stable squeezed states*' for the states introduced in (19), (22), (28) and (29) is suitable if one chooses the normally ordered Hamiltonian in the evolution operator.

Based on the nonlinear coherent states' formalism in [12], recently the '*nonlinear vacuum squeezed states*' have been introduced through the following actions on the vacuum states [11]:

$$S(\xi)|0\rangle = \exp\left[\tfrac{1}{2}\left(\xi A^{\dagger 2} - \xi^* B^2\right)\right]|0\rangle = |\xi, f\rangle, \tag{34}$$

$$\tilde{S}(\xi)|0\rangle = \exp\left[\tfrac{1}{2}\left(\xi B^{\dagger 2} - \xi^* A^2\right)\right]|0\rangle = |\widetilde{\xi, f}\rangle. \tag{35}$$

In the above equations, A and $A^\dagger$ may be obtained by simply setting $\alpha = 0$ in (6), similarly for B and $B^\dagger$. The number states expansion of the first one, $|\xi, f\rangle$ in (34), has the following superposition [11]:

$$|\xi, f\rangle = \mathcal{N} \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{n!} [f(2n)]! \left[\frac{\exp({\rm i}\phi)\tanh r}{2}\right]^n |2n\rangle, \tag{36}$$

where $\xi = \tanh r \exp(\mathrm{i}\phi)$ and $\mathcal{N}$ is chosen so that the states be normalized. The authors have studied the statistical properties of the squeezed states (36) for a special case with the nonlinearity function describing the centre of mass motion of a trapped ion (TI):

$$f_{\mathrm{TI}}(n) = L_n^1(\eta^2)\left[(n+1)L_n^0(\eta^2)\right]^{-1}, \tag{37}$$

where η is the Lamb–Dicke parameter and $L_m^n(x)$ are associated Laguerre polynomials. It can be easily investigated that the presented formalism recovers the results of [11] as a special case. Taking f to be a real function, i.e. setting $\alpha = 0$ in (18) (or $e_n = nf^2(n)$ in (19)), one eventually arrives at the nonlinear squeezed states in (36). Also, it is notable that setting $\alpha = 0$ in (21) (or $e_n = nf^2(n)$ in (22)) yields the '*dual family of nonlinear squeezed states*' in (36) as

$$|\widetilde{\xi, f}\rangle = \mathcal{N}' \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{n!} \frac{1}{[f(2n)]!} \left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n |2n\rangle. \tag{38}$$

The latter states are the number states expansion of (35). The states obtained in (36) and (38) are exactly equations (11*a*) and (11*b*) of a recent paper, respectively [15].

Moreover, it ought to be mentioned here that the constructed squeezed states in this paper, which have been called the 'Gazeau–Klauder squeezed states', do not fully guarantee the criteria of Gazeau and Klauder [8]. This is the fact that might be expected. Relaxing from the 'action identity' criteria (which imposed on the Gazeau–Klauder coherent states in order to emphasize on the 'classicality' of states) is neither necessary nor suitable here because the squeezed states are not essentially expected to show classical exhibition. Rather, generally most interesting in constructing the squeezed states is the nonclassical nature of them.

4. Gazeau–Klauder squeezed states of some physical systems

As some physical appearance of the proposed formalism, it is now possible to apply the scheme to a few well-known systems, i.e. simple harmonic oscillator, Pöschl–Teller and the infinite square-well potentials, hydrogen-like spectrum and at last the centre of mass motion of a trapped ion. The Gazeau–Klauder coherent states and the corresponding dual pairs of all these systems (except the last one) have been previously constructed [19].

Example 1. *Harmonic oscillator*

As the simplest example, one can apply the formalism to the harmonic oscillator Hamiltonian, whose nonlinearity function is equal to 1; hence, $\varepsilon_n = n = e_n$ and so $\mu(n) = n! = \rho(n)$. Note that we have considered a shifted Hamiltonian to lower the ground states energy to zero ($e_0 = 0 = \varepsilon_0$). Eventually, it can be easily observed that for the case of a harmonic oscillator, all the four classes of the Gazeau–Klauder squeezed states coincide with each other in the following way:

$$\begin{aligned}|\xi, \alpha, f\rangle &= \mathcal{N} \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{n!} \mathrm{e}^{-\mathrm{i}\alpha 2n} \left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n |2n\rangle \\ &= |\widetilde{\xi, \alpha, f}\rangle = |\xi, \alpha, \widetilde{f}\rangle = |\widetilde{\xi, \alpha, \widetilde{f}}\rangle. \end{aligned} \tag{39}$$

For these states, the normalization constant can be evaluated in a closed form as follows: $\mathcal{N} = (\cosh r)^{-\frac{1}{2}}$. Relation (39) clearly illustrates the '*self-duality*' of the Gazeau–Klauder squeezed states of the harmonic oscillator. Ordinarily the self-duality, which holds in this case, can be viewed as a checkpoint to be sure about the presented formalism [18, 19]. Note that substituting $\alpha = 0$ in (39) will recover the exact form of the squeezed vacuum obtained by the unitary operator $S(\xi)$ in (2). Strictly speaking, comparing $|\xi\rangle$ in (2) and $|\xi,\alpha\rangle$ in (39), it can be easily observed that ξ maps to $\xi\exp(-2\mathrm{i}\alpha)$, both in the complex plane, by the Gazeau and Klauder approach.

Example 2. *Pöschl–Teller and infinite square-well potentials*

These potentials and their coherent states are interesting due to various applications in many fields of physics such as atomic and molecular physics. The Gazeau–Klauder coherent states, corresponding to the Pöschl–Teller potential, have been demonstrated by Antoine *et al* in [3]. Their obtained results are based on the eigenvalues

$$e_n = n(n+\nu), \qquad \nu > 2. \tag{40}$$

In (40) $\nu = \lambda + \kappa$, where λ and κ are two parameters that determine the form (i.e. depth and width) of the potential well. Consequently, using the presented formalism the explicit form of the four classes of the Gazeau–Klauder squeezed states read as

$$|\xi,\alpha,f\rangle = \mathcal{N}\sum_{n=0}^{\infty} \mathrm{e}^{-\mathrm{i}\alpha 2n(2n+\nu)}\frac{(2n)!}{n!}\frac{\sqrt{[2n(2n+\nu)]!}}{n!}\left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n|2n\rangle. \tag{41}$$

$$\widetilde{|\xi,\alpha,f\rangle} = \mathcal{N}'\sum_{n=0}^{\infty} \mathrm{e}^{-\mathrm{i}\alpha 2n(2n+\nu)}\frac{(2n)!}{n!}\frac{(2n)!}{n!\sqrt{[2n(2n+\nu)]!}}\left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n|2n\rangle. \tag{42}$$

$$|\xi,\alpha,\tilde{f}\rangle = \tilde{\mathcal{N}}\sum_{n=0}^{\infty} \mathrm{e}^{-\mathrm{i}\alpha\frac{2n}{2n+\nu}}\frac{1}{n!}\sqrt{\left[\frac{2n}{2n+\nu}\right]!}\left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n|2n\rangle. \tag{43}$$

$$\widetilde{|\xi,\alpha,\tilde{f}\rangle} = \tilde{\mathcal{N}}'\sum_{n=0}^{\infty} \mathrm{e}^{-\mathrm{i}\alpha\frac{2n}{2n+\nu}}\frac{(2n)!}{n!}\sqrt{\left[\frac{2n+\nu}{2n}\right]!}\left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n|2n\rangle. \tag{44}$$

In the same manner, the squeezed states associated with the infinite square-well potential may be obtained by replacing $\nu = 2$ in equations (41)–(44).

Example 3. *Hydrogen-like spectrum*

We now choose the hydrogen-like spectrum whose corresponding coherent states have been a long-standing subject and discussed frequently in the literature. The eigenvalues of the one-dimensional model of such a system with the Hamiltonian $\hat{H} = -\omega/(\hat{n}+1)^2$ has been considered in [8] with eigenvalues

$$e_n = 1 - \frac{1}{(n+1)^2}, \tag{45}$$

to be such that $e_0 = 0$ (while $\omega = 1$). So, the Gazeau–Klauder squeezed states for this system can be easily calculated as

$$|\xi,\alpha,f\rangle = \mathcal{N}\sum_{n=0}^{\infty} \mathrm{e}^{-\mathrm{i}\alpha\frac{2n(2n+2)}{(2n+1)^2}}\frac{1}{n!}\sqrt{\left[\frac{2n(2n+2)}{(2n+1)^2}\right]!}\left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n|2n\rangle. \tag{46}$$

$$\widetilde{|\xi,\alpha,f\rangle} = \mathcal{N}'\sum_{n=0}^{\infty} \mathrm{e}^{-\mathrm{i}\alpha\frac{2n(2n+2)}{(2n+1)^2}}\frac{(2n)!}{n!}\sqrt{\left[\frac{(2n+1)^2}{2n(2n+2)}\right]!}\left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n|2n\rangle. \tag{47}$$

$$|\xi,\alpha,\tilde{f}\rangle = \tilde{\mathcal{N}}\sum_{n=0}^{\infty} \mathrm{e}^{-\mathrm{i}\alpha\frac{2n(2n+1)^2}{2n+2}}\frac{1}{n!}\sqrt{\left[\frac{2n(2n+1)^2}{2n+2}\right]!}\left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n|2n\rangle. \tag{48}$$

$$|\widetilde{\xi, \alpha, \widetilde{f}}\rangle = \widetilde{\mathcal{N}}' \sum_{n=0}^{\infty} \mathrm{e}^{-\mathrm{i}\alpha \frac{2n(2n+1)^2}{2n+2}} \frac{(2n)!}{n!} \sqrt{\left[\frac{2n+2}{2n(2n+1)^2}\right]!} \left[\frac{\exp(\mathrm{i}\phi)\tanh r}{2}\right]^n |2n\rangle. \tag{49}$$

Seemingly, this is the first time that the squeezed states associated with the hydrogen-like atom are introduced in such a direct relation to the general structure of the squeezed states and also to the related spectrum.

Example 4. *Centre of mass motion of a trapped ion*

As a final example, the centre of mass motion of a trapped ion with the nonlinearity function in (37) will be considered here. The associated (nonlinear) coherent and squeezed states were of much interest in the past decade [13, 14]. Fortunately, the presented formalism in section 2 allows one to define a $\hat{n}$-dependent Hamiltonian associated with the trapped ion system such that

$$\hat{H}_{\mathrm{TI}} = \hat{n} f_{\mathrm{TI}}^2(\hat{n}) = \frac{\hat{n}}{(\hat{n}+1)^2}\left[\frac{L_{\hat{n}}^1(\eta^2)}{L_{\hat{n}}^0(\eta^2)}\right]^2. \tag{50}$$

It seems that the Gazeau–Klauder type of squeezed states is also possible to be introduced if one considers the system with the n-dependent Hamiltonian as stated in (50). Therefore, the system will be specified with the spectrum

$$e_n = \frac{n}{(n+1)^2}\left[\frac{L_n^1(\eta^2)}{L_n^0(\eta^2)}\right]^2, \tag{51}$$

from which the spectrum of the dual system will be easily calculated using $\varepsilon_n = \frac{n^2}{e_n}$. Hence, having e_{2n} and ε_{2n}, the four classes of the Gazeau–Klauder squeezed states for the centre of mass motion of the trapped ion may be easily obtained with the help of the general structure introduced in equations (19), (22), (28) and (29).

5. The quantum statistical properties and squeezing of Gazeau–Klauder squeezed states

The quantum statistical properties of the squeezed states outlined in the present paper as well as the squeezing exhibition of them will be considered in this section. As one of the manifest nonclassicality features of all the generalized squeezed states obtained in this paper, one may refer to the '*oscillatory number distribution*' of these states. Photon statistics of the states in (19), (22), (28) and (29) may be easily calculated in general forms

$$P(2n) = \mathcal{N}^2 \frac{[e_{2n}]!}{(n!)^2}\left[\frac{\tanh r}{2}\right]^{2n}, \tag{52}$$

$$P'(2n) = \mathcal{N}'^2 \left[\frac{(2n)!}{n!}\right]^2 \frac{1}{[e_{2n}]!}\left[\frac{\tanh r}{2}\right]^{2n}, \tag{53}$$

$$\widetilde{P}(2n) = \widetilde{\mathcal{N}}^2 \frac{[\varepsilon_{2n}]!}{(n!)^2}\left[\frac{\tanh r}{2}\right]^{2n}, \tag{54}$$

$$\widetilde{P}'(2n) = \widetilde{\mathcal{N}}'^2 \left[\frac{(2n)!}{n!}\right]^2 \frac{1}{[\varepsilon_{2n}]!}\left[\frac{\tanh r}{2}\right]^{2n}, \tag{55}$$

respectively. $\mathcal{N}, \mathcal{N}', \widetilde{\mathcal{N}}$ and $\widetilde{\mathcal{N}}'$ in the above four relations are determined in (20), (23), (20) and (23), respectively. Generally, it can be seen that for all of the introduced squeezed states, one has

$$\mathbf{P}(2n) \neq 0 \qquad \text{while} \quad \mathbf{P}(2n+1) = 0 \qquad \text{for all} \quad n, \tag{56}$$

which clearly shows the nonclassicality nature of the obtained states in (19), (22), (28) and (29).

To complete the study of the statistical properties of the squeezed states associated with the physical examples introduced in the previous section, the Mandel parameter and the squeezing of the quadratures of the field will be illustrated numerically since the analytical form of the above quantities cannot be given in a closed form. The calculation of the Mandel parameter, defined as

$$Q = \frac{\langle \hat{n}^2\rangle - \langle \hat{n}\rangle^2}{\langle \hat{n}\rangle} - 1, \qquad \hat{n} = a^\dagger a, \tag{57}$$

determines the super-Poissonian (if $Q > 0$), sub-Poissonian (if $Q < 0$) and Poissonian (if $Q = 0$) nature of the states. The case of the Poissonian is the characteristics of the standard coherent states. Sub-Poissonian is an important property which implies the nonclassicality of the states, and the super-Poissonian statistics has important consequence for the properties of localization and temporal stability of the wave packet [3]. Further, for the squeezing of the states according to $x = (a + a^\dagger)/\sqrt{2}$, $p = (a - a^\dagger)/(\mathrm{i}\sqrt{2})$, one has to calculate the variances

$$(\triangle p)^2 = \langle p^2\rangle - \langle p\rangle^2 = \langle a^\dagger a\rangle - \tfrac{1}{2}\langle a^2\rangle - \tfrac{1}{2}\langle (a^\dagger)^2\rangle + \tfrac{1}{2}, \tag{58}$$

$$(\triangle x)^2 = \langle x^2\rangle - \langle x\rangle^2 = \langle a^\dagger a\rangle + \tfrac{1}{2}\langle a^2\rangle + \tfrac{1}{2}\langle (a^\dagger)^2\rangle + \tfrac{1}{2}. \tag{59}$$

In the latter two equations the equalities $\langle a\rangle = 0 = \langle (a^\dagger)\rangle$ have been used, which hold for all classes of the obtained squeezed states (since all of them are some superpositions of the even Fock states, $|2n\rangle$). Evidently, all of the expectation values in equations (57)–(59) must be calculated with respect to the squeezed states introduced in (19), (22), (28) and (29) for the quantum physical examples outlined in section 4. Squeezing holds in the x-, p-quadrature if $(\triangle x)^2$, $(\triangle p)^2$ be less than $\frac{1}{2}$, respectively. Seemingly, it will be enough to bring here the terms necessary for one of the four classes of the Gazeau–Klauder squeezed states; other cases may be derived in a similar fashion, straightforwardly. For example, for the states in (19), it can be easily observed that

$$\langle (a^\dagger a)\rangle = 2\mathcal{N}^2 \sum_{n=0}^{\infty} \frac{[e_{2n+2}]!}{n!(n+1)!}\left(\frac{\tanh r}{2}\right)^{2n+2}, \tag{60}$$

$$\langle (a^\dagger a)^2\rangle = 4\mathcal{N}^2 \sum_{n=0}^{\infty} \frac{[e_{2n+2}]!}{[(n+1)!]^2}\left(\frac{\tanh r}{2}\right)^{2n+2}, \tag{61}$$

$$\langle a^2\rangle = \mathcal{N}^2 \sum_{n=0}^{\infty} \mathrm{e}^{\mathrm{i}\alpha(e_{2n}-e_{2n+2})} \frac{\sqrt{(2n+1)(2n+2)[e_{2n+2}]![e_{2n}]!}}{n!(n+1)!}\,\mathrm{e}^{-\mathrm{i}\phi}\left(\frac{\tanh r}{2}\right)^{2n+1}, \tag{62}$$

$$\langle (a^\dagger)^2\rangle = \langle a^2\rangle^\dagger. \tag{63}$$

Setting $\phi = 0$ and the eigenvalues e_{2n} associated with all the physical examples of section 4, Q, $(\triangle x)^2$ and $(\triangle p)^2$ can be readily evaluated. Similar calculations can be done for the states in (22), (28) and (29), straightforwardly.

Part of the results of numerical calculations, for some classes of the squeezed states, has been presented in figures which follows. Figure 1(a) shows the super-Poissonian statistics for the harmonic oscillator for all values of r (note that the results for any state of the harmonic oscillator cover all the four classes of the Gazeau–Klauder squeezed states for harmonic oscillator due to its self-duality). For the trapped ion system, the states in (19) show the sub-Poissonian statistics for a wide range of values of r and η (figure 1(b) shows Q as a function

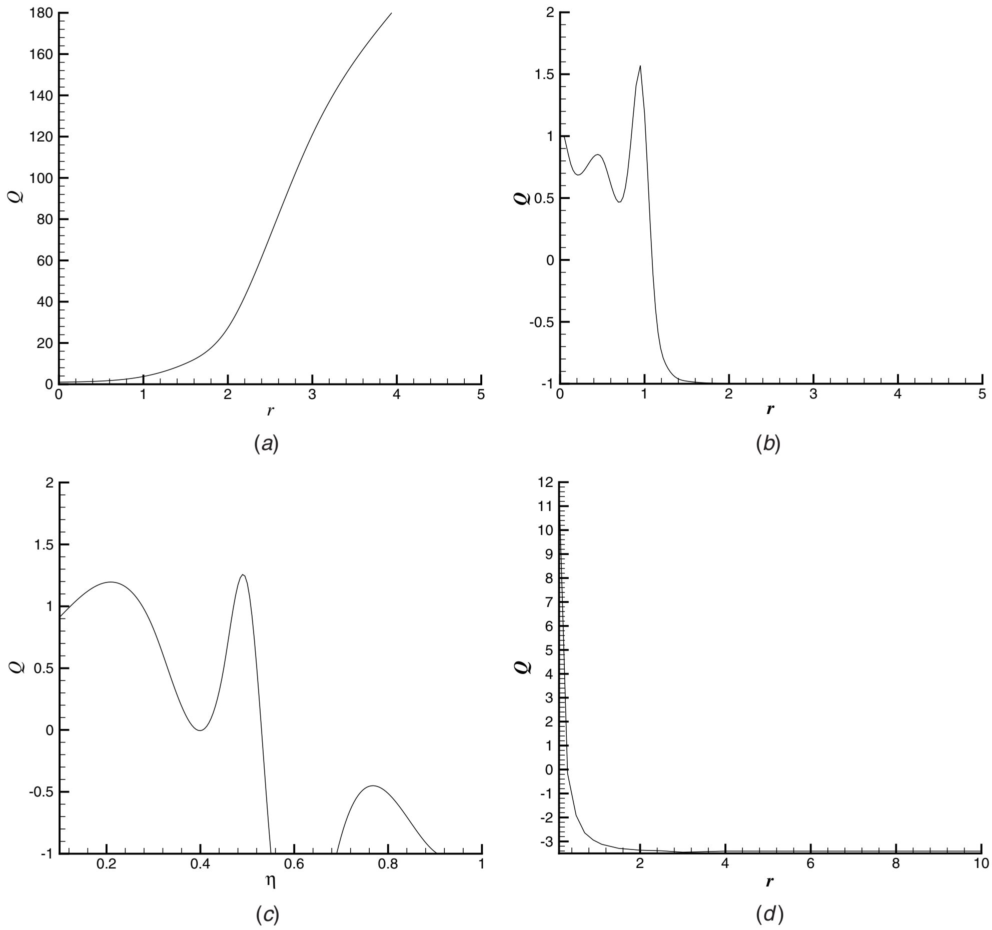

Figure 1. (*a*) The Mandel parameter of the squeezed states of the harmonic oscillator as a function of r. (*b*) The Mandel parameter of a trapped ion's squeezed states of equation (19) as a function of r ($\eta = 0.5$). (*c*) The Mandel parameter of a trapped ion's squeezed states of equation (19) as a function of η ($r = 1$). (*d*) The Mandel parameter of a trapped ion's squeezed states of equation (29) as a function of r ($\eta = 0.7$).

of r for a fixed value of $\eta = 0.5$ and figure 1(*c*) shows Q versus η for a fixed value of $r = 1$). Figure 1(*d*) describes Q versus r with the choice $\eta = 0.7$ for the trapped ion corresponding to formula (29). It illustrates the super-Poissonian exhibition of the constructed squeezed states in a wide range of the values of r.

The numerical calculations show that the Gazeau–Klauder type of squeezed states of trapped ion motion according to equation (22) is super-Poissonian, using the parameters $r \geqslant 0.1, \eta = 0.7$, and the same system according to equation (28) has $Q < 0$, for $r \geqslant 0.03, \eta = 0.7$. The Mandel parameter for infinite square-well and Pöschl-Teller potentials gives $Q < 0$ when equation (19) has been used, $Q > 0$ for equation (22), $Q > 0$ while equation (28) has been in consideration and $Q < 0$ for equation (29). It is worth noting that generally all of the sub-Poissonian cases whose results have been given here without the graphs have $Q > 0$ very near to $r \simeq 0$. The calculations of the Mandel parameter Q for

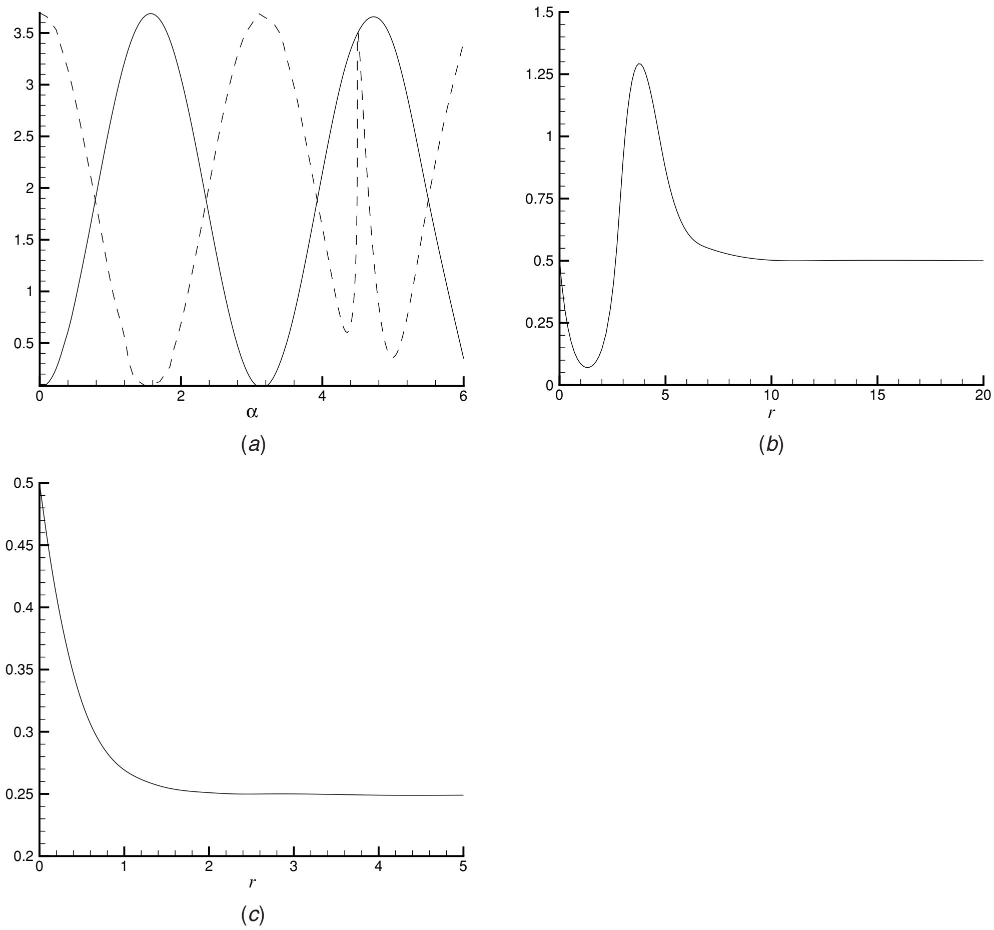

Figure 2. (*a*) The squeezing in x-quadrature (solid line) and p-quadrature (dashed line) for the harmonic oscillator's squeezed states as a function of α ($r = 1$). (*b*) Plot of $(\triangle x)^2$ as a function of r for the squeezed states of the harmonic oscillator ($\alpha = 1.5$). (*c*) Plot of $(\triangle p)^2$ as a function of r ($\alpha = 0.5$) of the hydrogen-like atom's squeezed states, where the states of equation (19) have been used.

the hydrogen atom show $Q > 0$ for equation (19), $Q < 0$ when equation (22) has been used, $Q < 0$ for equation (28) and $Q > 0$ when equation (29) has been used.

The plots of squeezing in x- and p-quadrature for the harmonic oscillator have been shown in figure 2(*a*) in terms of α for the fixed value of $r = 1$. Squeezing can be observed in x-quadrature, when for example $1.22 \leqslant \alpha \leqslant 1.92$ and $4.36 \leqslant \alpha \leqslant 5.064$, and also in p-quadrature in two distinct regions, when $0 \leqslant \alpha \leqslant 0.35$ and $2.8 \leqslant \alpha \leqslant 3.49$. Again, for the harmonic oscillator $(\triangle x)^2$ is plotted as a function of r for a fixed value of $\alpha = 1.5$ in figure 2(*b*). It is observed that the squeezing occurs for $r \leqslant 2.6$ in x-quadrature. Figure 2(*c*) indicates the squeezing in p-quadrature for the hydrogen atom according to formula (19) (when $\alpha = 1.5$) which occurs for all values of r. Figure 3(*a*) demonstrates the squeezing in p-quadrature of the Pöschl–Teller ($\nu = 5$ and $\alpha = 4$) and infinite square-well potentials ($\nu = 2$ and $\alpha = 0.1$), when equation (22) has been used. In this case, the squeezing

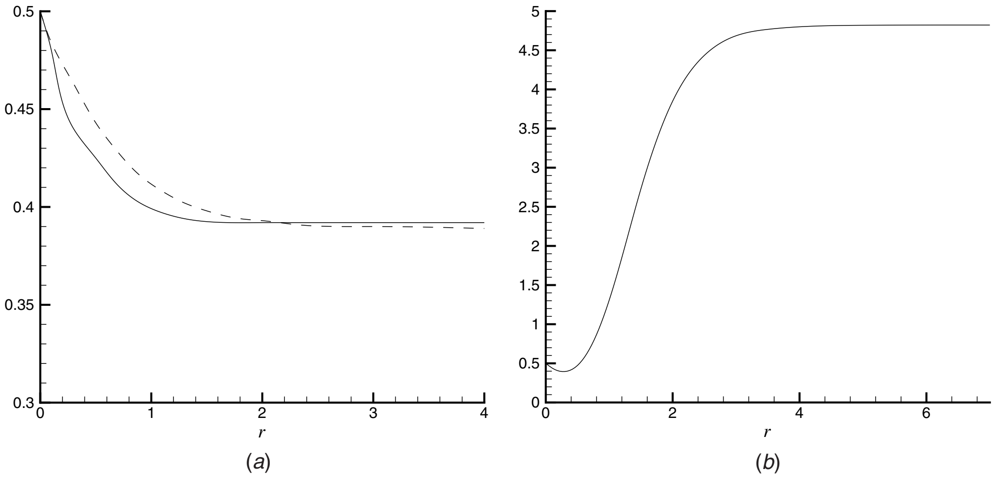

Figure 3. (*a*) Plots of $(\triangle p)^2$ as a function of r for the square-well potential, $\nu = 2$ (solid line) and Pöschl–Teller potential, $\nu = 5$ (dashed line). In these cases, the structure of equation (22) is considered. (*b*) Plot of $(\triangle x)^2$ of the trapped ion's squeezed states of equation (22) as a function of r ($\alpha = 1.5, \eta = 0.1$).

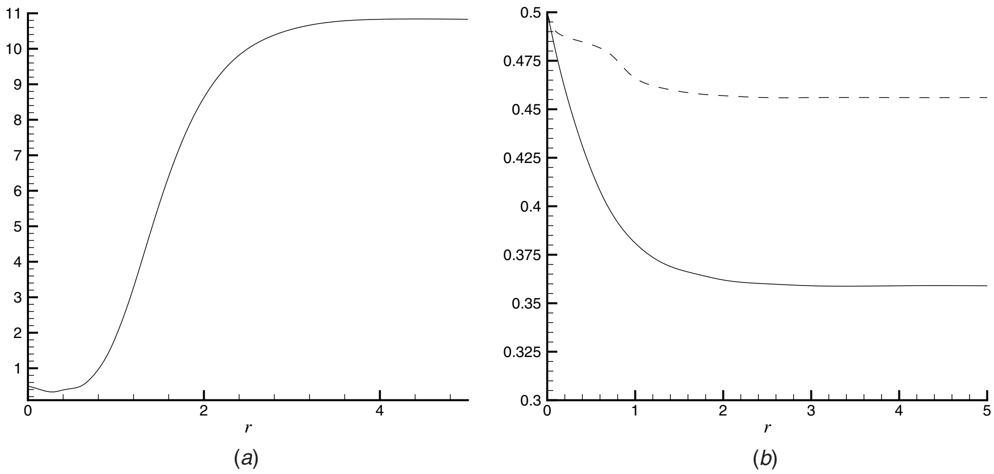

Figure 4. (*a*) Plot of $(\triangle x)^2$ of the squeezed states of the trapped ion as a function of r ($\alpha = 1.5, \eta = 0.3$), when equation (28) is used. (*b*) Plots of $(\triangle p)^2$ as a function of r for the square-well potential ($\nu = 2$) and Pöschl–Teller potential ($\nu = 20$), where equation (28) is used with $\alpha = 1.5$.

may interpolate between x- and p-quadrature by tuning α and r parameters. In figure 3(*b*), $(\triangle x)^2$ for the trapped ion in the form of the states (22) has been plotted, the squeezing of which may be observed in the range $r \leqslant 0.5$ (the other parameters are chosen so that $\eta = 0.1$ and $\alpha = 1.5$). In figure 4(*a*) the squeezing in x-quadrature for the trapped ion in the form of the states (28) is plotted (using the parameters $\eta = 0.3$ and $\alpha = 1.5$). As can be observed, squeezing occurs in the range $r \leqslant 0.6$. Figure 4(*b*) indicates the squeezing in p-quadrature of the Pöschl–Teller ($\nu = 20$) and infinite square-well ($\nu = 2$) potentials, where

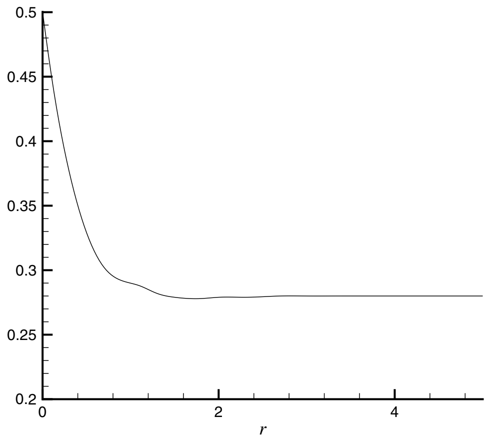

Figure 5. Plot of $(\triangle p)^2$ as a function of r for the hydrogen-like atom's squeezed states, where equation (29) with $\alpha = 1.5$ is used.

in both cases the α parameter is chosen to be 1.5. It is interesting to note that squeezing occurs in the whole range of r for the two cases, and the one for the infinite square-well potential is always stronger. At last, the numerical calculations for the hydrogen atom in the form of the states (29) show that the p-quadrature is squeezed for all values of r, when $\alpha = 1.5$ (figure 5). Altogether, by the above results the nonclassicality nature of the introduced squeezed states in this paper has been established, obviously.

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